Problem A

The Mystery of Magnetic Train

Abstract

Based on the basic electromagnetism model, this paper gives a sufficiently persuasive explanation of the mechanism of the "magnetic train" and analyzes several relevant parameters that affect the train's speed and power. Depending on the law of electromagnetic induction and the distribution of generated magnetic field of solenoid， we build three equivalent models to explain the phenomenon from different perspectives. These models can explain the train's motion. By means of theoretical deduction and numerical analysis based on the law of Biot-savart's law and Ampere's circuital theorem of magnetic field in a straight solenoid, we conducted a detailed theoretical calculation on the forces that the generated magnet act on the train. Then we discussed several possible relevant factors that affect the train's motion. Finally, we drew some conclusion that explain the influence and discussed about lifting up the efficiency of magnetic train system.

Key words: solenoid; magnetic induction; magnetic distribution; Matlab simulation

1. Introduction

Using only a small cylindrical battery, two groups of button magnets, and some copper coil wire, one can easily make a moving “train”. Both ends of battery need to be connected with button magnets (by magnetic force only). Put the connection into the copper coil and immediately the train starts to travel through the copper “track”. If the copper coil is end to end, the train will go over and over without stopping. The whole movement is amazing for the train’s continuing its fast moving through a long tunnel. Therefore, some questions are raised up naturally that what the mechanism behind all this is, where the force that pushing the magnetic train forwards comes from, what factors can affect the train’ motion and how about their influence degree is.

Some discussion of this phenomenon can be seen on the Internet. People tried different ways to give their explanation but their theories are insufficient in science. One article gives a relatively reasonable explanation and analysis (Stephen Irons,2015) [1] but still lacks the quantitative analysis.

In this paper, we focus our study on how the phenomenon occurs by analyzing the internal relation between the composition of this tiny moving system. Based on some basic electromagnetism theories and laws, we established three models to explain this amazing phenomenon from different perspective. Then we based on these models, established a quantitative analysis model to investigate relevant parameters which affect the train’s speed and power. Finally, we found out what the relevant factors are and how their impacts on the train’s motion.

2. Assumption

In this paper, we treat all the compositions of “train” system as ideal components. It means that the electromagnetic characteristics of battery, strong magnets and copper coil can be simplified, which helps our analysis.

For discussing the reason why the magnetic train moves that way, we set the movement happening in a linear track. As for other types of track, we believe that they can be divided into several linear tracks. Hence this simplification makes no differences under different circumstance.

The establishment time of an electromagnetic field (EMF) is close to the speed of light while the motion of magnetic train is macro low-speed, hence we take the magnetic field as constant in dt and neglect the establishment time of it. Besides these, for fully simplifying these problems, we neglect other environmental factors and set all the experiments condition the best.

3. Theory and model

3.1 A qualitative explanation of the train's movement

Newton's First Law states that an object will remain at rest or in uniform motion in a straight line unless acted upon by an external force. Our goal is to figure out what the external force is and how does it occur in magnetic train system.

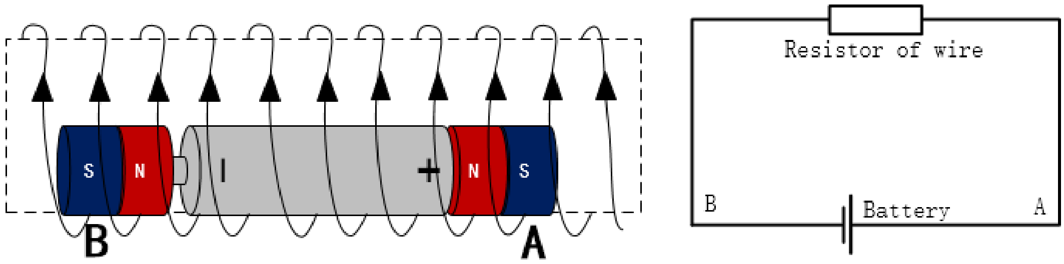


Fig. 1. The structure of magnetic train and its equivalent current circuit.

As illustrated in Figure 1, two magnets attached to both ends of battery get to the copper wire, so the battery makes a circuit with magnets' connection. The current starts from the battery, and flows through the magnet and copper wire, and ends at the battery.

We take the section of the copper coil out and treat it as a finitely long solenoid. As Ampere's law suggested, we know that the current generates a magnetic field in the solenoid. And the distribution of magnetic induction line is shown in Figure 2 from which we can see the magnetic field in solenoid is strong and uniform while the field outside is weak and divergent.

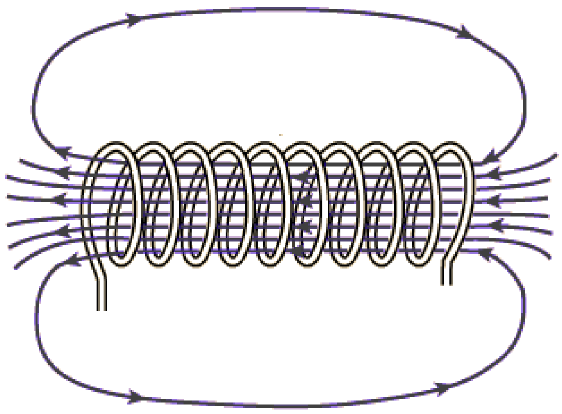


Fig. 2. The magnetic distribution in solenoid.

The magnetic field generated by current can be equivalent to a bar magnet. If the button magnets are set in the way shown in Figure 3, the left magnet gets pulled and the right one gets pushed (Magnets or currents of the same polarity will repel each other, while those of different polarity will attract each other). Therefore, the connection body will move leftwards.

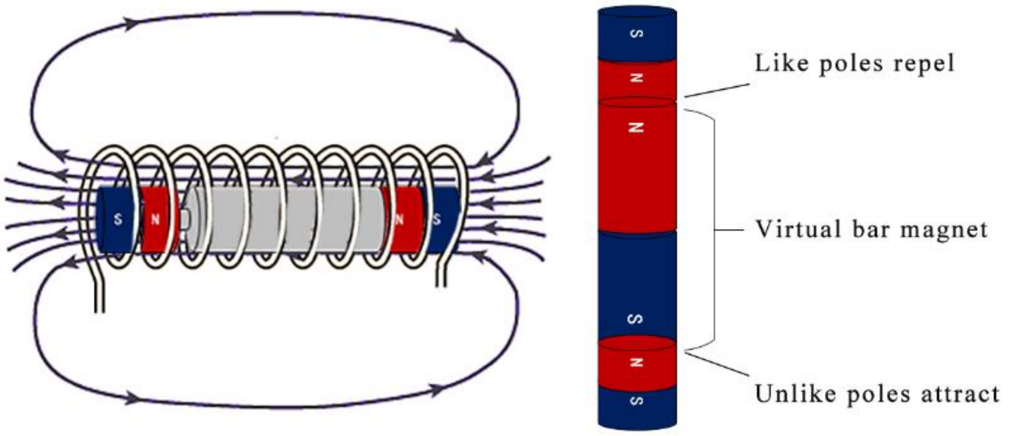
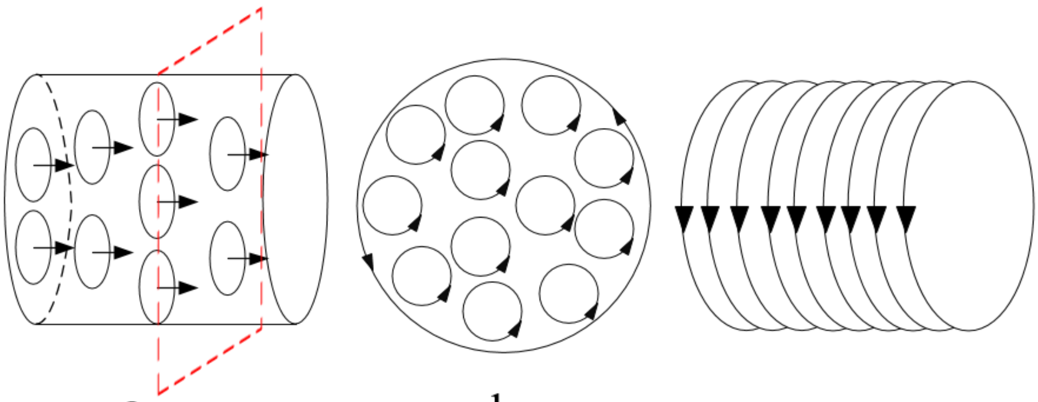


Fig. 3. The virtual bar magnet and its effect on both button magnets.

Treating the long solenoid as a bar magnet is a simple way to explain the movement. But the attraction force and repulsion force is hard to define where they come from. Here we introduced another equivalent model. According to Ampere's molecular current hypothesis[2], every magnet molecular of normal iron is equal to ring current. The orientations of these molecular magnet moments are disordered while in magnetic iron the magnet moments are in the same direction (Figure 4. a) and directions of adjacent molecular currents are in opposite (Figure 4. b). Hence under the surface of magnet, the molecular currents countervail and only the surface remains the molecular current. Magnet can be considered as a composition of multiple loops of homodromous molecular current on the surface of magnet. Therefore, we can treat a magnet as a solenoid with electric current (Figure 4. c).



a b c

Fig. 4. The molecular current model and the equivalent model of a magnet.

To analyze the force that molecular current sustains, we use Matlab to simulate the magnetic field (the number of turns is set as 6) [3]. From Figure 5, we know that the specific distribution of magnet line. Inner the solenoid, the lines are paralleled with each other and in both ends of the solenoid, the lines diverge outside, which is quite similar to our qualitative analysis in the virtual bar magnet model.

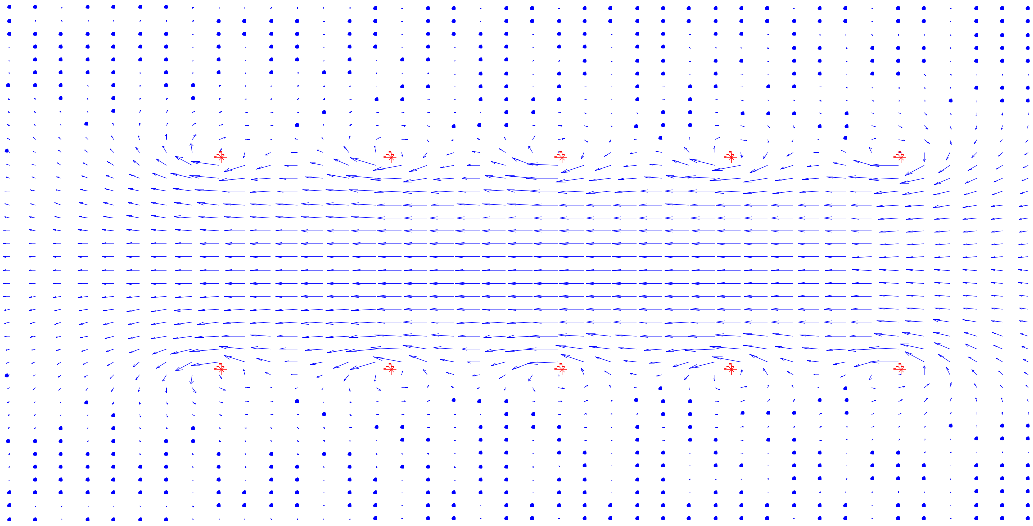


Fig. 5. The accurate magnetic distribution described in Matlab.

Then the progress of analyzing becomes easy and plain. We chose a ring current from the equivalent solenoid of each magnet and we conducted force analysis on it. Like shown in Figure 6, the left ring current $d\vec{I_1}$ is in a magnetic field $\vec{B_1}$ and the resultant force of this current ($d\vec{F_{11}} = d\vec{I_1} \times \vec{B_1}$) points to the left (Fleming's rule). The same situation sees on the right ring current. So we have two resultant forces $\vec{F_1}$ and $\vec{F_2}$ pointing to the left. And two magnets can be divided into several such a current couple, hence the resultant force of magnets couple points to the left. Then the magnetic train moves leftwards. As the train moves, the generated magnetic field moves with the same direction and speeds. As long as the train's head is in the copper coil, the movement will not stop.

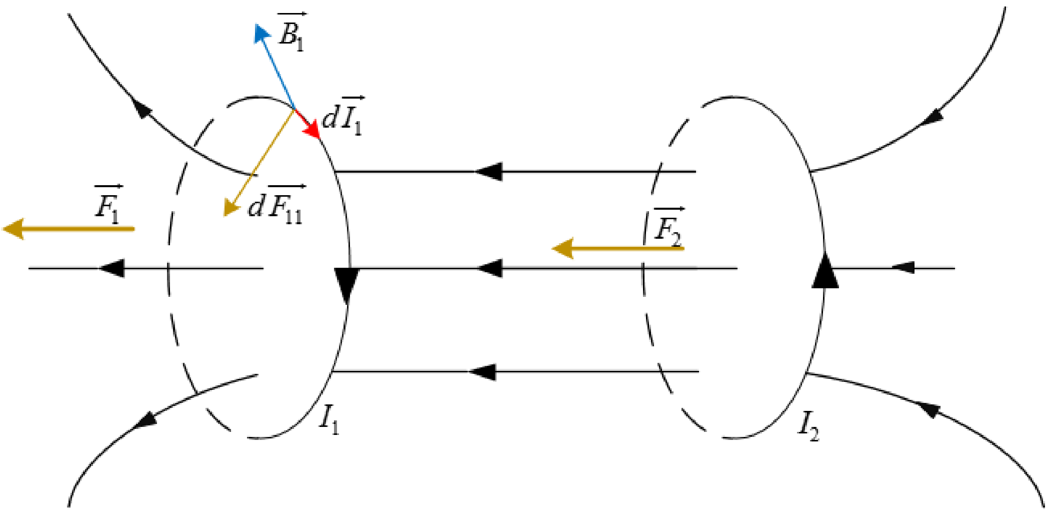


Fig. 6. The force analysis based on the equivalent solenoid model of magnets.

Till now, the Ampere's molecular current hypothesis hasn't been proved wrong, at least in physics experiments. However, cutting a permanent magnet into several tiny ring currents is far from acceptable for common people. Here we can introduce another approach to explain the origin of the forces without discussing these molecular currents from the microcosmic point of view.

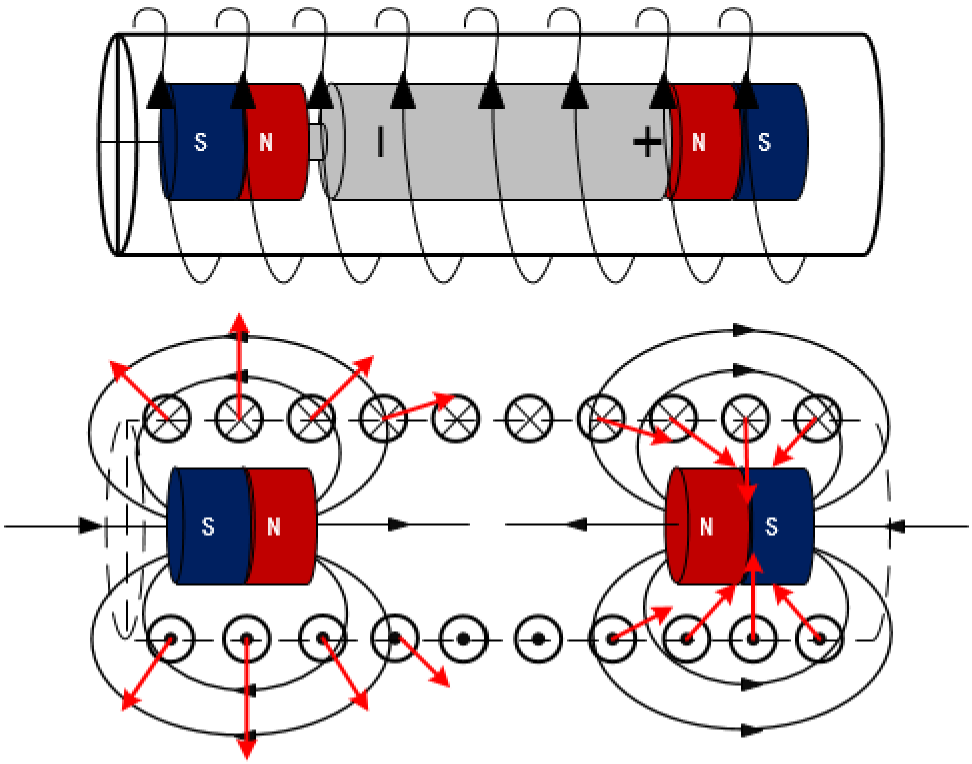


Fig. 7. The force analysis for solenoid in the magnetic field of button magnets.

As the Figure 7 illustrates, we take the infinite solenoid with electric current as our research object and remove the dry battery. The solenoid is in the magnetic field of the two button magnets. We drew the general distribution of magnet lines, and conducted the force analysis for copper wires. The results show clearly that the resultant force of copper wires points to the right. In terms of Newton's third law states, all forces between two objects exist in equal magnitude and opposite direction[4]. Therefore, the magnet filed generated in copper wire act the opposite force pointing to the left, which is the direction that the magnetic train moves to.

3.2 An quantitative analysis of the train's movement

We pick the section of copper coil which includes the entire body of magnetic train (Figure 8).

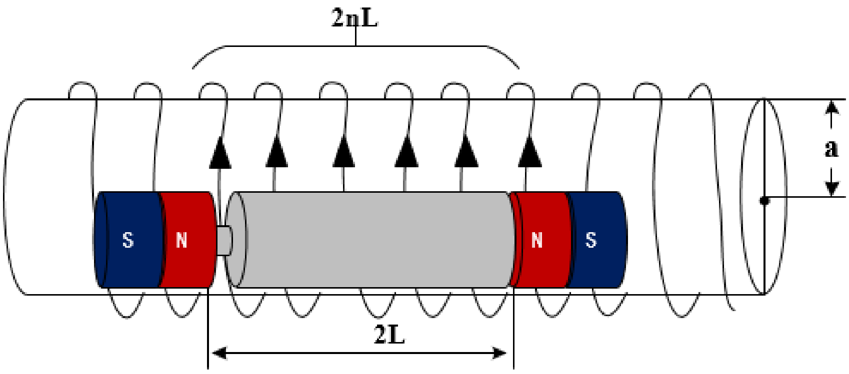


Fig. 8. One section of the long copper solenoid.

As mentioned before, the two magnets need to connect with the copper wire to form a current circuit. The length of wire is

$$L_{wire} = 4\pi anL$$

where R is the radius of solenoid, n is the number of turns of solenoid, and L is the length of solenoid.

Then we have the resister of wire is

$$R_{wire} = \frac{1}{\sigma L_{wire}}$$

where σ is the electric conductivity of copper wire (5.8×10^{7} S/m).

The circuit current is

$$I = \frac{E}{R_{wire}} = \sigma L_{wire} E = 4n\pi\sigma aLE$$

where E is the electromotive force of battery.

Then we consider generated magnetic field distribution:

Based on Biot-savart's law, the current element $I'd\vec{l}'$ (located at $\vec{r}'$)'s contribution to magnetic field (at $\vec{r}$)is :

$$d\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{I'd\vec{l}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

where μ_0 is the magnetic permeability in vacuum and $\vec{r}$ is the vector pointing from current element to any position in the magnetic field.
If the current element is at the origin point ($\vec{r}' = 0$), then

$$d\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{I'd\vec{l}' \times \hat{r}}{r^2}$$

Then we consider such magnetic field distribution in solenoid (Figure 9): in terms of the symmetry properties of solenoid, once we have the distribution situation of magnetic field in yoz plane, the distribution situation of the entire space can be derived.

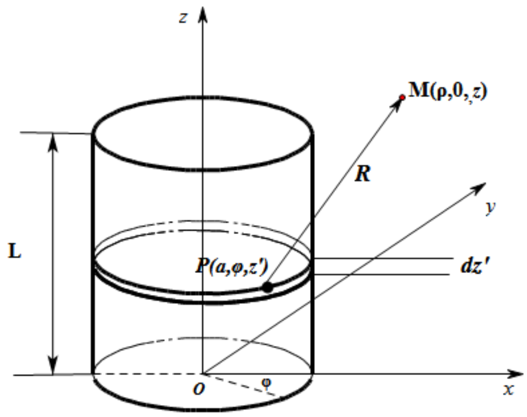


Fig. 9. Geometry analysis under cylindrical coordinate.

Take a slice (thickness is dz) of the cylinder along the **z** axis. Then the ring current is $I' = Indz$. Current element $I'd\vec{l}'$ is on this slice, which located at point $P(a, \varphi, z')$ (in cylindrical coordinates). Pick one point $M(\rho, 0, z)$ in this system.

According to the geometry relations in Figure 9, we have:

$$d\vec{l}' = \hat{\varphi} a d\varphi = a d\varphi(-\hat{x}\sin\varphi + \hat{y}\cos\varphi)$$

$$|\vec{r} - \vec{r}'| = \sqrt{(z - z')^2 + \rho^2 + a^2 - 2a\rho\cos\varphi}$$

From this we can roll out magnetic vector:

$$d\vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\int_l \frac{I'd\vec{l}'}{|\vec{r}-\vec{r}'|}=\frac{\mu_0 I'}{4\pi}\int_0^{2\pi}\frac{a(-\hat{x}\sin\varphi+\hat{y}\cos\varphi)}{\sqrt{(z-z')^2+\rho^2+a^2-2a\rho\cos\varphi}}d\varphi$$

In this equation:

$$\int_0^{2\pi}\frac{-\hat{x}\sin\varphi}{\sqrt{(z-z')^2+\rho^2+a^2-2a\rho\cos\varphi}}d\varphi=\hat{x}\frac{-1}{a\rho}\sqrt{(z-z')^2+\rho^2+a^2-2a\rho\cos\varphi}\Big|_{(z-z')^2+\rho^2+a^2-2a\rho}^{(z-z')^2+\rho^2+a^2-2a\rho}=0$$

Hence,

$$d\vec{A}(\vec{r})=\frac{\mu_0 I'a}{4\pi}\int_0^{2\pi}\hat{y}\frac{\cos\varphi}{\sqrt{(z-z')^2+\rho^2+a^2-2a\rho\cos\varphi}}d\varphi$$

In cylindrical coordinates, $\hat{y}=\hat{\varphi}$. So when it comes to $M(\rho,0,z)$,there exists

$$d\vec{A}(\vec{r})=\hat{\varphi}\frac{\mu_0 I'a}{4\pi}\int_0^{2\pi}\frac{\cos\varphi}{\sqrt{(z-z')^2+\rho^2+a^2-2a\rho\cos\varphi}}d\varphi=\hat{\varphi}\frac{\mu_0 I'a}{2\pi}\int_0^{\pi}\frac{\cos\varphi}{\sqrt{(z-z')^2+\rho^2+a^2-2a\rho\cos\varphi}}d\varphi$$

According to rotation formula,

$$d\vec{B}=\nabla\times d\vec{A}=-\hat{\rho}\frac{\partial dA}{\partial z}+\hat{z}\frac{1}{\rho}\frac{\partial(\rho dA)}{\partial\rho}$$

Generate it into the formula $\hat{y}=\hat{\varphi}$, we have:

$$\begin{cases} dB_\rho=-\dfrac{\partial dA}{\partial z}=\dfrac{\mu_0 I'a(z-z')}{2\pi}\displaystyle\int_0^{\pi}\frac{\cos\varphi}{((z-z')^2+\rho^2+a^2-2a\rho\cos\varphi)^{\frac{3}{2}}}d\varphi \\ dB_z=\dfrac{1}{\rho}\dfrac{\partial(\rho dA)}{\partial\rho}=\dfrac{dA}{\rho}+\dfrac{\mu_0 I'a}{2\pi}\displaystyle\int_0^{\pi}\frac{(a\cos\varphi-\rho)\cos\varphi}{((z-z')^2+\rho^2+a^2-2a\rho\cos\varphi)^{\frac{3}{2}}}d\varphi \end{cases}$$

So

$$\begin{cases} A=\displaystyle\int_{2L}dA=\frac{\mu_0 nIa}{2\pi}\int_0^{\pi}\int_{-L}^{L}\frac{\cos\varphi dz'}{\sqrt{(z-z')^2+\rho^2+a^2-2a\rho\cos\varphi}}d\varphi \\ B_\rho=\displaystyle\int_{2L}dB_\rho=\frac{\mu_0 nIa}{2\pi}\int_0^{\pi}\int_{-L}^{L}\frac{(z-z')\cos\varphi dz'}{((z-z')^2+\rho^2+a^2-2a\rho\cos\varphi)^{\frac{3}{2}}}d\varphi \\ B_z=\displaystyle\int_{2L}dB_z=\frac{A}{\rho}+\frac{\mu_0 nIa}{2\pi}\int_0^{\pi}\int_{-L}^{L}\frac{(a\cos\varphi-\rho)\cos\varphi dz'}{((z-z')^2+\rho^2+a^2-2a\rho\cos\varphi)^{\frac{3}{2}}}d\varphi \end{cases}$$

Let $R^2=\rho^2+a^2-2a\rho\cos\varphi$, then integrate over dz', we can get

$$\begin{cases} A = \dfrac{\mu_0 nIa}{2\pi}\displaystyle\int_0^\pi \ln\frac{\sqrt{(z+L)^2+R^2}+z+L}{\sqrt{(z-L)^2+R^2}+z-L}\cos\varphi d\varphi \\ B_\rho = \dfrac{\mu_0 nIa}{2\pi}\displaystyle\int_0^\pi \left[\frac{\cos\varphi}{\sqrt{(z-L)^2+R^2}} - \frac{\cos\varphi}{\sqrt{(z+L)^2+R^2}}\right] d\varphi \\ B_z = \dfrac{A}{\rho} + \dfrac{\mu_0 nIa}{2\pi}\displaystyle\int_0^\pi \frac{(a\cos\varphi-\rho)}{R^2}\left[\frac{z+L}{\sqrt{(z+L)^2+R^2}} - \frac{z-L}{\sqrt{(z-L)^2+R^2}}\right]\cos\varphi d\varphi \end{cases}$$

According to relevant solution in reference [5],

$$A = \frac{\mu_0 nI}{2\pi}\int_0^\pi \frac{a^2\rho\cos^2\varphi}{R^2}\left[\frac{z+L}{\sqrt{(z+L)^2+R^2}} - \frac{z-L}{\sqrt{(z-L)^2+R^2}}\right] d\varphi$$

We can get the Magnetic induction caused by solenoid of every place in the space. For example, at point $M(\rho, 0, z)$

$$\begin{cases} B_\rho = \dfrac{\mu_0 nIa}{2\pi}\displaystyle\int_0^\pi \left[\frac{\cos\varphi}{\sqrt{(z-L)^2+R^2}} - \frac{\cos\varphi}{\sqrt{(z+L)^2+R^2}}\right] d\varphi \\ B_z = \dfrac{\mu_0 nIa}{2\pi}\displaystyle\int_0^\pi \frac{a-\rho\cos\varphi}{R^2}\left[\frac{z+L}{\sqrt{(z+L)^2+R^2}} - \frac{z-L}{\sqrt{(z-L)^2+R^2}}\right] d\varphi \end{cases}$$

We use numerical methods to seek distribution of magnetic induction solenoid. in order to reduce the number of calculations and ensure sufficient accuracy, we use a high precision complex algebraic Gauss quadrature formula [6].

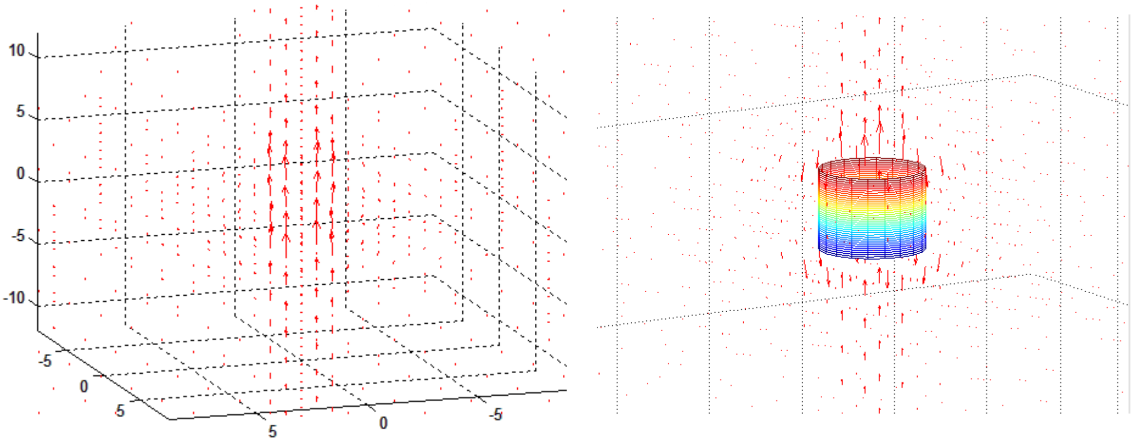


Fig. 10. The magnetic induction distribution (in vector expression).

The result of integration can be visualized in figure above. In the right figure, colorful cylinder stands for the solenoid. And the red vectors are the magnetic induction.

After having the numeric solution of magnet distribution, we can continue calculating the forces on the two button magnets:

$$d\vec{F}_m = X_0 \vec{H}\, \nabla \bullet \vec{H} dm \approx \frac{X_0 \nabla \bullet \vec{B}}{\mu_0^{\ 2}} \vec{B} dm$$

In this equation, X_0 is magnetic susceptibility (m³/kg),which means the force a unit mass of ore particles suffered in a unit external magnetic field. M means mass(kg). The mass density of button magnet is $dm = \delta dV$. So the push force these two magnets get from the field is

$$\vec{F}_m = \int_{2V} d\vec{F} = \int_{2V} \frac{X_0 \nabla \bullet \vec{B}}{\mu_0^{\ 2}} \vec{B} \delta dV$$

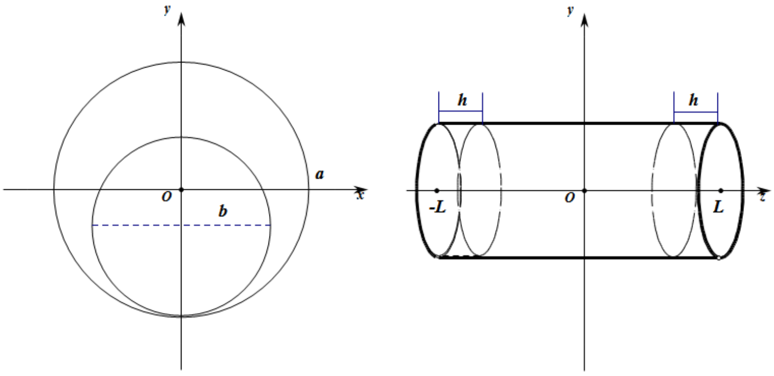


Fig. 11. The integration area of resultant force.

The place where the button magnets lie is determined by (Figure 11)

$$(x-a+b)^2 + y^2 \le b^2 (L \le z \le L+h, -L+h \le z \le -L)$$

The resultant force the train undertakes is

$$\vec{F} = \vec{F}_m - \eta S = \int_{2V} \frac{X_0 \nabla \bullet \vec{B}}{\mu_0^{\ 2}} \vec{B} \delta dV - \eta S$$

where η is the friction factor. S is the contact area.

According to the the law of conservation of energy, we have

$$E = E_k + E_m + E_e = E_{e0}$$

where E_k is the kinetic energy of the connection body of battery and button magnets, E_m is the magnetic energy of the system, E_e is the electronic energy of the dry battery and E_{e0} is the initial electronic energy stored in the battery.

To take the connection body as our research object, then its total energy $E_c = E_k + E_e$.

Then we have

$$P_c = \frac{dE_c}{dt} = -\eta mgv + P_e - I^2 R = -\eta mg \frac{P_c}{F} + P_e - I^2 R$$

Where F is the resultant force of this moving system and P_e is the power of generating current from battery (a chemistry progress)

Then we can get

$$P_c = \frac{F\left(P_e - I^2 R\right)}{\eta mg}, v = \frac{P_c}{F} = \frac{\left(P_e - I^2 R\right)}{\eta mg}$$

4. Results and discussion

As we can see from the formula before, in a short time, the P_e and I remains the same level as initial value. But with hours of service going, the battery will lose its power (like the reduction curve in Figure 12) [7], and the speed will get lower.

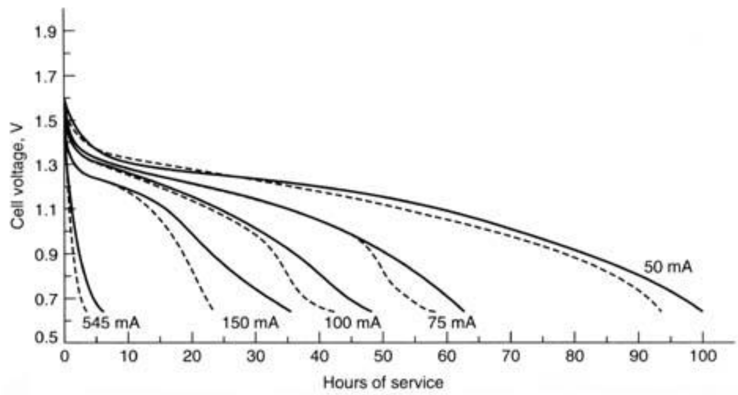


Fig.12. The reduction curve of different dry battery with the time going.

Besides, the friction is also a direct influential factor. In the moving device, the bottom of the two button magnets contacts directly with the copper wire. Although the both surfaces are similar to be smooth, friction is still cannot be neglected for the fluctuations of the copper wire.

In terms of the deducted formula, the power of the motion system is affected by the battery, friction coefficient, the mass of the system itself and also the force that the system undertakes. As we calculated before, that resultant force is determined by many factors like the thickness of two button magnets, the length of copper wire, etc. Take the length of copper wires in the circuit (also the length of battery) as our research object, we calculated the resultant force under different copper wire. From the Figure 13, we can see the force raises up with the wire length increases in general tendency. Actually, in terms of circumstance, the length of battery is fixed.

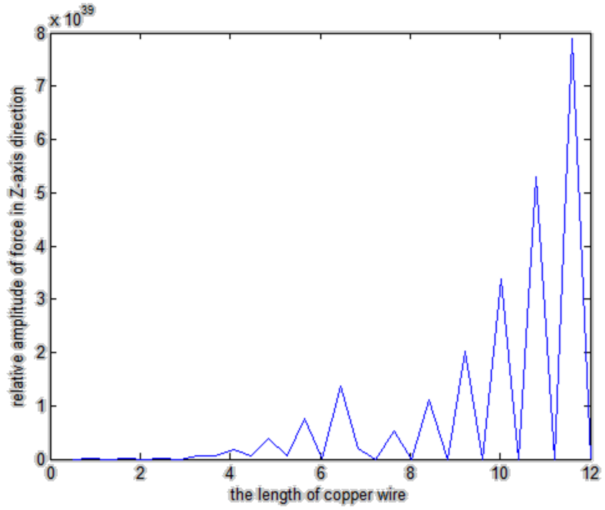


Fig.13. The influence that the length of copper wire on the resultant force.

From all the theory deduction and numeric simulation on these problems, we can draw a series conclusion. The magnetic field interacting with the magnet's original field is generated by the current. The energy provided by the battery is the original power of this "train".

Though the dream to create a perpetual motion machine has been proved to be impossible and opposite to the basic theory and laws of physic, we still hope to invent or make progress to enhance the efficiency of machines. In this mini system, we can decrease the energy waste by resistance and friction means increasing the efficiency. There are various practicable ways. Decrease the contact area or use button magnets whose surface are smoother can decrease the friction consume. In addition, using button magnets with bigger magnetic susceptibility is also useful.

5.Strengths and Weaknesses

5.1Advantages

We established three reasonable qualitative models to explain the working mechanism of magnetic train. Both the models come from the most basic electromagnetism theories and laws. The virtual bar magnet model treats the long solenoid as a magnet, which is easy to understand and acceptable but doesn't give a specific account on the origin of forces of attraction and repulsion. To be more persuasive in theory, we analyze the problem from the micro perspective, basing on the Ampere's molecular current hypothesis. After detailed force analysis on both button magnets, we found out the source of pushing and pulling force and gave graphic description.

To consider the effect of relevant parameters on the train's speed and power, we tried to established a quantitative model to calculate the train's speed and power.

5.2 Weaknesses

When calculating both the speed and power, we gave some variables some possible constant value. Therefore, the robustness of our model is unknown. The influence degree of each possible relevant parameter may be affected by the constant value we set.
Also, as a result of limitation of traditional integration analysis, we haven't solved the entire analytical solutions as final answers. So we use numeric solutions to qualitatively analyses the relevant parameters and their effect on the magnetic train's speed and power. For this situation, traditional mathematical methods show their restriction. More scientific solution should be studied by using some analyzing and simulating software like Maxwell, COMSOL Multiphysics, etc.
Besides, when analyzing the influence of different parameters on the train's movement, we consider them respectively and the general analyzing is insufficient. Because the inner relations among these factors are difficult to conduct an overall analysis.

6. Reference

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[7] Figure: http://www.sithtech.net/photographyzto/Lithium-vs-Alkaline-Battery-Life.html

Appendix I

Matlab code:

```
% Describe the magnetic distribution in solenoid
clear
rh=2.5;i0=10;mu0=4*pi*1e-7;n=10 %set a series of parameters
m=(n+1)/2
xmax=6;ymax=6;ngrid=40;
cx(1:ngrid,1:ngrid)=zeros;cy(1:ngrid,1:ngrid)=zeros;
c0=mu0/4*pi;
nh=20;
ngrid1=nh+1;
xmax1=0;
ymax1=2*pi;
xplot=linspace(- xmax,ymax,ngrid); %set the 2-dimensions grid
yplot=linspace(- xmax,ymax,ngrid);
theta0=linspace(0,2*pi,21);
theta1=theta0(1:nh);
y1=rh*cos(theta1);
z1=rh*sin(theta1);
theta2=theta0(2:nh+1);
y2=rh*cos(theta2);
z2=rh*sin(theta2);
dlx=0;dly=y2- y1;dlz=z2- z1;
xc=[- (n- 1)/2:2:(n- 1)/2];yc=(y2+y1)/2;zc=(z2+z1)/2;
for k=1:m
for i=1:ngrid
for j=1:ngrid
rx=xplot(j)- xc(k); %to calculate every grid point to the
current element in x direction
ry=yplot(i)- yc;
rz=0- zc;
r3=sqrt(rx.^2+ry.^2+rz.^2).^3;
dlxr_x=dly.*rz- dlz.*ry;
dlxr_y=dlz.*rx- dlx.*rz;
bx(i,j)=sum(c0*i0*dlxr_x./r3); by(i,j)=sum(c0*i0*dlxr_y./r3);
end
end
cx(1:ngrid,1:ngrid)=cx(1:ngrid,1:ngrid)+bx(1:ngrid,1:ngrid);
cy(1:ngrid,1:ngrid)=cy(1:ngrid,1:ngrid)+by(1:ngrid,1:ngrid);
end
quiver(xplot,yplot,cx,cy); %to describe the vector field
plot(xc,rh,' r*' )
```

```
hold on
plot(xc,- rh,' r*' )
```

```
%%
clear;clc
%I = U/(2*pi*a);
mu0 = 4*pi*1e-7;
I = 1; %
n = 1; %
a = 0.5; %
L = 1; %

C = mu0*n*I*a/2/pi; %

xmax=9;
ymax=9;
zmax=20;
ngrid=50;
x=linspace(- xmax,xmax,ngrid); %
y=linspace(- ymax,ymax,ngrid);
z=linspace(- zmax,zmax,ngrid);

fai = linspace(0,pi,40);
cx=zeros(ngrid,ngrid,ngrid);%
cy=zeros(ngrid,ngrid,ngrid);
cz=zeros(ngrid,ngrid,ngrid);

b = 10^10;  %

for i=1:ngrid      %
    for j=1:ngrid
        for k=1:ngrid
            rou = sqrt( x(i)^2 + y(j)^2 );
            Bp  =  C*sum(      cos(fai).*  (      1./sqrt(   (z(k)-
L)^2+rou^2+a^2-2*a*rou*cos(fai)                    )             -
1./sqrt(  (z(k)+L)^2+rou^2+a^2-2*a*rou*cos(fai)  )   )   );
            Bx  =  Bp*x(i)/rou;
            By  =  Bp*y(j)/rou;
            Bz   =  C*sum(      (    (a-rou*cos(fai))./  (rou^2+a^2-
2*a*rou*cos(fai))  )  .* ...
                 (      (z(k)+L)./sqrt(      (z(k)-L)^2+rou^2+a^2-
```

```
2*a*rou*cos(fai) ) - (z(k)-L)./sqrt( (z(k)+L)^2+rou^2+a^2-
2*a*rou*cos(fai) ) ) );

          cx(i,j,k) = b*Bx ;
          cy(i,j,k) = b*By ;
          cz(i,j,k) = b*Bz ;
       end
    end
end

 [xplot,yplot,zplot] = ndgrid(linspace(- zmax,zmax,ngrid));
scale =1;
quiver3(xplot,yplot,zplot,cx,cy,cz,scale,'r');
axis([-xmax xmax -ymax ymax -12 12 ])
view([40,120,40]);
% view([0,90,90]);
hold on

[r,zz]=ndgrid((0:.05:1)*2*pi,-1:.05:1);
 x=cos(r);
 y=sin(r);
 h1=mesh(x,y,zz);

x0 = 0.0001;
density = 1;
h = 0.3;
b = 0.25;

div = divergence(cx,cy,cz);
Fz = density*x0*div/mu0^2.* cz;


F = 0;
for i=1:ngrid
    for j=1:ngrid
        for k=1:ngrid
            if (L<=z(k)&&z(k)<=(L+h))||((-L+h)<=z(k)&&z(k)<=-L)
                if ((x(i)-a+b)^2 + y(j)^2) <= b^2
                    %cz(i,j,k)
                    F = F+ abs(Fz(i,j,k));
                end
            end
        end
    end
```

```
end
```